**CS 2510**

LAST NAME \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_FIRST NAME \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Homework #3**

**PROBLEM 1** [points:24]

Find values

1. 27 mod 6 = 3

1. -27 mod 6 = 3

1. 44 mod 9 = 8

1. -44 mod 9 = 1

1. ⌊−4.37 + ⌈−6.73⌉⌋ + ⌈−6.73⌉ = -17

1. ⌈−6.7 − 4.7⌉ = -11

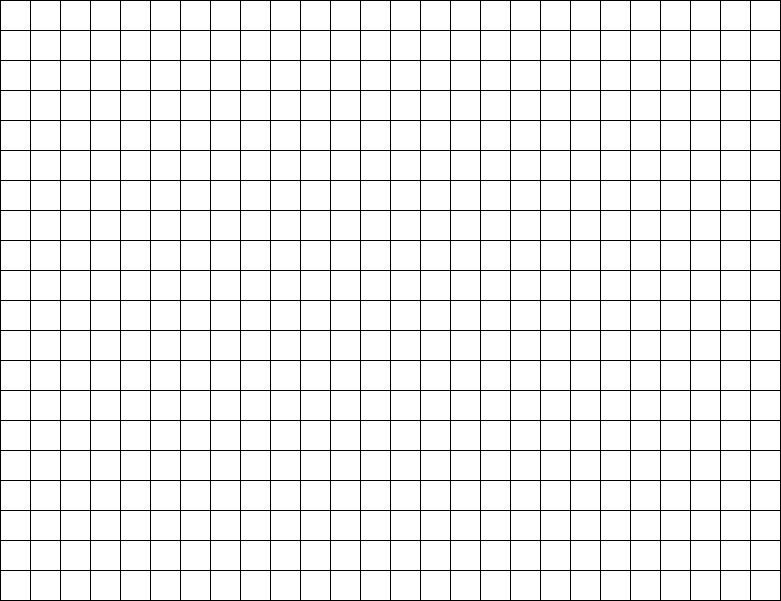
1. ⌈−6⌉ = -6

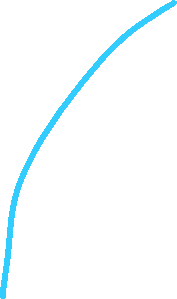
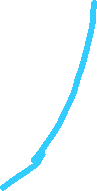
# h) ⌊−6⌋ = -6

**PROBLEM 2** [points:26] Consider the function 𝑓(𝑥) = 𝑥 − 1

𝑥−1

* Draw a graph [5pts]





* Find/define the domain and the range of *f*. [6 points; 3 pts for each]

Domain: Range:

* Is *f* one-to-one? Why? [5 points]

Yes, it is 1-to-1:

By the horizontal line test, the function is intersected at 2 place for every given y. This is a one-to-one function.

* Is *f* onto? Explain. [5 points]

Yes, f(x) is onto because there is a value of x where f(x) = y. All the values of x are included in the domain. Every value in the range corresponds to a specific value in the domain.

* Is *f* bijective? [2 points]

Yes, the function is bijective, since it is both one-to-one and onto.

* Calculate the values of *f* at *x*=0, *x*=0.5, and *x*=2. [3 points]

**PROBLEM 3** [points:10]

Find a Big-Oh upper bound for the following function

# 𝑓

Reason your answer (explain what you disregard, etc).

Since is of a lower order than , it can be disregarded as a constant. is a polynomial with the highest degree of a and some constants. Let C be a constant and N\_0 be another constant. Since the 968 is another constant, it can be disregarded. Now will be bound by an exponent higher than it. The nearest exponent larger than 1.5 is 2. Let there be a function , where C is a constant greater than or equal to 44. The upper bound for the function is .

**PROBLEM 4** [points:20]

Determine whether each of these functions = 𝑂(𝑥3).

*Justify* your answer (also can use table at the end of the slides for help).

# a) 𝑓(𝑥) = 𝑥3 + 33: Yes, this is 𝑂(𝑥3), as f(x) is of a lower order than . It occurs before the 𝑂(𝑥3) function in the table.

1. 𝑓; Yes, this is 𝑂(𝑥3), as f(x) is of a lower order than . It occurs before the 𝑂(𝑥3) function in the table.
2. 𝑓; Yes, this is an n log(n) function, which is of a lower order than . It occurs before the 𝑂(𝑥3) function in the table.
3. 𝑓(𝑥) = 44𝑥!; No, f(x) is of a higher order than . It occurs after the 𝑂(𝑥3) function in the table.

# e) 𝑓(𝑥) = 3𝑥; No, f(x) is of a higher order than . It occurs after the 𝑂(𝑥3) function in the table.

**PROBLEM 5** [points:20]

Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

1. the function that assigns to each nonnegative integer its last digit

Domain is all the natural numbers from 1 to : {1,2,3,4,5,6,7…..} = N

Range is numbers from 0 to 9: {0,1,2,3,4,5,6,7,8,9}

1. the function that assigns the next largest integer to a positive integer

Domain is the set of all positive integers: {1,2,3,4,5,6,7…..}.

Range is all numbers greater than the smallest positive integer: {2,3,4,5,6,7…..}

1. the function that assigns to a bit string the number of bits in the string

Domain is the set of all bit string.

Range is the set of all-natural numbers.

1. the function that assigns to a bit string the number of one bits in the string

Domain is the set of all bit strings.

Range is the set of all-natural numbers.

**Extra Credit PROBLEM** [points: 5]

Prove or disprove that for all real numbers x and y, ⌊𝑥 + 𝑦⌋ = ⌊𝑥⌋ + ⌊𝑦⌋.

This equation is not true for all real numbers x and y.

Let x=y=0.5

⌊0.5 + 0.5⌋ = ⌊0.5⌋ + ⌊0.5⌋

⌊1⌋ ⌊0.5⌋ + ⌊0.5⌋

1 0